| Centre Number | | | | | | Candidate Number | | | | |
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| For Examiner's Use | |
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| Examiner's Initials | |
| Question | Mark |
| 1 | |
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| TOTAL | |

General Certificate of Education
Advanced Level Examination
June 2012

# Mathematics MFP3

## Unit Further Pure 3

**Thursday 14 June 2012 9.00 am to 10.30 am**

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

**1** The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = \sqrt{(2x)} + \sqrt{y}$$

and $$y(2) = 9$$

Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = h\mathrm{f}(x_r, y_r)$ and $k_2 = h\mathrm{f}(x_r + h, y_r + k_1)$ and $h = 0.25$, to obtain an approximation to $y(2.25)$, giving your answer to two decimal places. *(5 marks)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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**2 (a)** Write down the expansion of $\sin 2x$ in ascending powers of $x$ up to and including the term in $x^5$. *(1 mark)*

**(b)** Show that, for some value of $k$,

$$\lim_{x \to 0} \left[ \frac{2x - \sin 2x}{x^2 \ln(1 + kx)} \right] = 16$$

and state this value of $k$. *(4 marks)*

QUESTION PART REFERENCE

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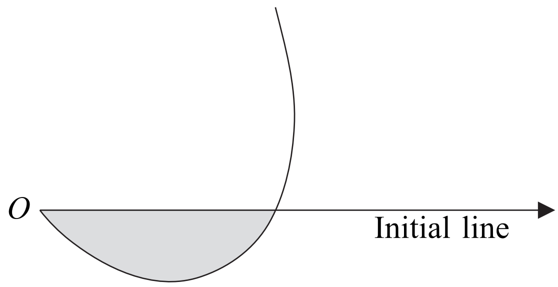
**3** The diagram shows a sketch of a curve $C$, the pole $O$ and the initial line.




The polar equation of $C$ is

$$r = 2\sqrt{1 + \tan\theta}, \qquad -\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{4}$$

Show that the area of the shaded region, bounded by the curve $C$ and the initial line, is $\dfrac{\pi}{2} - \ln 2$. *(4 marks)*

QUESTION PART REFERENCE

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**4 (a)** By using an integrating factor, find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{4}{2x+1}y = 4(2x+1)^5$$

giving your answer in the form $y = \mathrm{f}(x)$. *(7 marks)*

**(b)** The gradient of a curve at any point $(x, y)$ on the curve is given by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 4(2x+1)^5 - \frac{4}{2x+1}y$$

The point whose $x$-coordinate is zero is a stationary point of the curve. Using your answer to part **(a)**, find the equation of the curve. *(3 marks)*

QUESTION PART REFERENCE

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**5 (a)** Find $\int x^2 e^{-x}\, dx$. *(4 marks)*

**(b)** Hence evaluate $\int_0^{\infty} x^2 e^{-x}\, dx$, showing the limiting process used. *(3 marks)*

QUESTION PART REFERENCE

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**6** It is given that $y = \ln(1 + \sin x)$.

**(a)** Find $\dfrac{dy}{dx}$. *(2 marks)*

**(b)** Show that $\dfrac{d^2y}{dx^2} = -e^{-y}$. *(3 marks)*

**(c)** Express $\dfrac{d^4y}{dx^4}$ in terms of $\dfrac{dy}{dx}$ and $e^{-y}$. *(3 marks)*

**(d)** Hence, by using Maclaurin's theorem, find the first four non-zero terms in the expansion, in ascending powers of $x$, of $\ln(1 + \sin x)$. *(3 marks)*

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**7 (a)** Show that the substitution $x = \mathrm{e}^t$ transforms the differential equation

$$x^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 4x\frac{\mathrm{d}y}{\mathrm{d}x} + 6y = 3 + 20\sin(\ln x)$$

into
$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} - 5\frac{\mathrm{d}y}{\mathrm{d}t} + 6y = 3 + 20\sin t$$
*(7 marks)*

**(b)** Find the general solution of the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} - 5\frac{\mathrm{d}y}{\mathrm{d}t} + 6y = 3 + 20\sin t$$
*(11 marks)*

**(c)** Write down the general solution of the differential equation

$$x^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 4x\frac{\mathrm{d}y}{\mathrm{d}x} + 6y = 3 + 20\sin(\ln x)$$
*(1 mark)*

QUESTION PART REFERENCE

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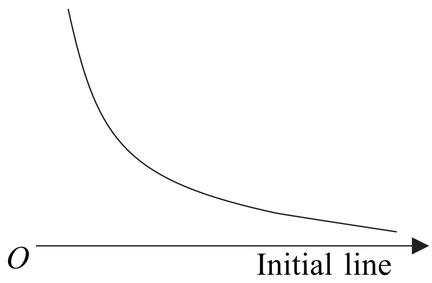
**8 (a)** A curve has cartesian equation $xy = 8$. Show that the polar equation of the curve is $r^2 = 16 \operatorname{cosec} 2\theta$. *(3 marks)*

**(b)** The diagram shows a sketch of the curve, $C$, whose polar equation is

$$r^2 = 16 \operatorname{cosec} 2\theta, \quad 0 < \theta < \frac{\pi}{2}$$




**(i)** Find the polar coordinates of the point $N$ which lies on the curve $C$ and is closest to the pole $O$. *(2 marks)*

**(ii)** The circle whose polar equation is $r = 4\sqrt{2}$ intersects the curve $C$ at the points $P$ and $Q$. Find, in an exact form, the polar coordinates of $P$ and $Q$. *(4 marks)*

**(iii)** The obtuse angle $PNQ$ is $\alpha$ radians. Find the value of $\alpha$, giving your answer to three significant figures. *(5 marks)*

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**END OF QUESTIONS**

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